Curve Fitting Of Cumulative Discovery and Production of Nigerian Petroleum Resources Using Composite Underground Reservoir

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Abstract – In this work, models were first developed from material balance of Nigerian petroleum around a composite reserve. Process control concepts were introduced to obtain the transfer functions so that as input functions are varied, new models are obtained, to note which input have the best impact. Hubbert oil depletion concept was employed for the peak determination. The Nigerian Petroleum Data were obtained from the Department of Petroleum Resources (DPR) of the Ministry of Petroleum and Minerals Resources, 7 Kofo Abayomi Street, Victoria Island, Lagos, as the experimental data for 57 years giving 57 data points. MatLab Package 7.9 version was employed in the mathematical computations and curve-fittings. From the curve-fitted plots, It is found that the Nigerian oil reserve will finish in the year 2682AD and the gas will follow sooth in the year 3151AD.

Index Terms – Curve-fitting, Cumulative discovery and production, Nigerian petroleum resources, peak, composite underground reservoir.

1. INTRODUCTION

The oil boom of early 1970s led Nigeria to neglect its strong agricultural and light manufacturing bases in favour of over dependence on crude oil. The country joined the organization of petroleum exporting countries (OPEC) in July 1971 as the eleventh member (NNPC, 1988). The first high increase in the price of crude oil was from 1973-1974 due to oil embargo by Arab countries. This yielded monumental financial benefits to the country (Adenikinju, 1996). Subsequently, crude oil continued to play predominant and more strategic roles in the economy.

By 2000, Nigeria's proven oil reserves were estimated to be 25 billion barrels (4km3), natural gas reserves were well over 2800km^3) (PPMC, 1994). As an OPEC member, in mid 2001 its crude oil production was averagely around 2.2 million barrels (350.000m3) per day. As the reserves continued to increase, both due to new discoveries and politics, her OPEC quota continues to rise to the fifth largest OPEC producer of petroleum in the world.

The Nigerian Economy as at today (2017) is about 90% dependent on petroleum, to the detriment of other sectors because of false assumptions that petroleum will be forever, and that its depletion theory is unreal. However, contrary to this erroneous view, Nigerian petroleum is fast depleting at the rate to be determined in this research work.

Against this background, the problem is to find when Nigerian Petroleum will peak or had peaked in the past years, so that we can monitor the downward bumps of plateau to exhaustion. Since oil formation gives rise to gas formation or vice versa the exhaustion of one leads to the eventual exhaustion of the other with time. And of this, Nigerian petroleum depletion profiles, peaking and exhaustion dates, this study is poised to model and forecast.

2. NATURAL RESOURCES DEPLETION MODEL DEVELOPMENT

2.1 Background

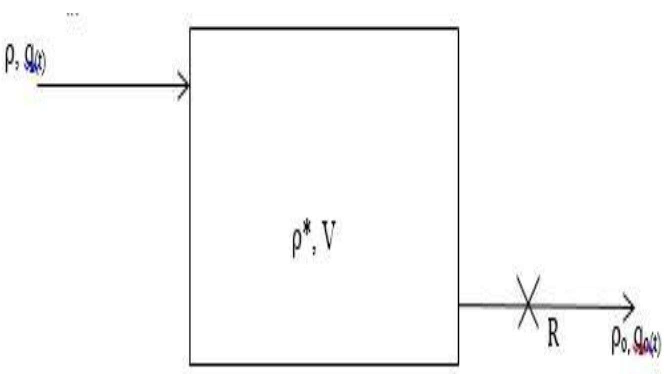
Generally, the trend for most natural resources depletion is first increasing return, followed by a constant return and finally diminishing return. If there is no corresponding replenishment of the produced (consumed) resources, the diminishing return will continue until the resource is finally exhausted or flatten. The law of diminishing return is hence applicable to natural resource depletion with general model given by the equation below;

$$P = C_1t - C_2t^2 \qquad (2.1)$$

2.2 Assumptions and Model Development for Curve-fitting

1. One composite oil well replacing all oil wells drilled by all the oil companies in Nigeria.
2. The data collected from (DPR) is the sum total of all the production, discovery and reserves from all the oil companies in Nigeria.
3. The composite oil well have one input and one output.
4. The material balance around the well will result in first order ODE.
5. The density of the crude entering the reserve, in the reserve and out from the reserve is the same.

6. At the beginning there is initial volume of reserve, V_O.
7. The nation's crude will finish when the rate of change of the reserve is zero (Hubbert concept, ie $\frac{dQ_R}{dt} = 0$)



Mass Balance

Mass flow in –Mass flow out = rate of accumulation of mass in the reservoir.

Mathematically,

$$\rho q - \rho_o q_o = \frac{d(\rho * V)}{dt} \qquad (2.2)$$

Assumptions:

- At the t = 0, volume is V_O
- Uniform density, i.e $\rho_o = \rho = \rho^*$
- Laminar flow, $q_o = \frac{V}{R}$

Eqn (1) becomes $q - \frac{V}{R} = \frac{dV}{dt}$

$$(2.3)$$

Taking Laplace of eqn (2.3), $Q_{(S)} - \frac{V(s)}{R} = sV(s) - V_O$

$$V(s) = \frac{Vo}{[S+\frac{1}{R}]} + \frac{Q(s)}{[S+\frac{1}{R}]} \qquad (2.4)$$

*If the input function is $Q_{(S)} = \frac{1}{S[S+\frac{1}{R}]}$, then eqn (2.4) is

$$V(s) = \frac{Vo}{[S+\frac{1}{R}]} + \frac{1}{S[S+\frac{1}{R}]^2} \qquad (2.5)$$

Taking the inverse Laplace of eqn (2.5) and simplifying gives;

$$V_{(t)} = F(R^2(1 - e^{-\frac{1}{R}t}) + (Vo - Rt)\, e^{-\frac{1}{R}t}) \qquad (2.6a)$$

On differentiation with respect to time, eqn (2.6a) which is a cumulative production-time history yields an annual production-history, egn (2.6b).

$$P_{(t)} = F(t - \frac{Vo}{R})e^{-\frac{1}{R}t} \qquad (2.6b)$$

At peak, $\frac{d^2V}{dt^2} = \frac{dP}{dt} = 0$; $Rt - R^2 - V_o = 0$

$$t_{pk} = R + \frac{Vo}{R} \qquad (2.6c)$$

*If the input function is $Q(s) = \frac{1}{S[S+\frac{1}{R}]^2}$, then eqn (2.4) is

$$V(s) = \frac{Vo}{[S+\frac{1}{R}]} + \frac{1}{S[S+\frac{1}{R}]^3} \qquad (2.7)$$

Taking inverse Laplace of eqn (2.7) and simplifying gives

$$V_{(t)} = F(R^3(1 - e^{-\frac{1}{R}t}) + (Vo - R^2t - \frac{Rt2}{2})\, e^{-\frac{1}{R}t}) \qquad (2.8a)$$

On differentiation with respect to time, eqn (2.8a) which is a cumulative production-time history yields an annual production-time history, eqn (2.8b)

$$P_{(t)} = F(\frac{t2}{2} - \frac{Vo}{R})e^{-\frac{1}{R}t} \qquad (2.8b)$$

At peak, $\frac{d^2V}{dt^2} = \frac{dP}{dt} = 0$; $Rt^2 - 2R^2t - 2Vo = 0$

$$t_{pk} = R \pm \frac{\sqrt{(R4 + 2RVo)}}{R} \qquad (2.8c)$$

The derivations continue as shown in the table 2.1

2.3 Cumulative Plots

In cumulative plots, the result is always sigmoidal in profile. The data for discovery is many times bigger than the data for production and so they are plotted separately. The combination plot shows the discovery profile pushing production profile down to almost align with zero x-axis.

The difference between their y-axis values is the cumulative reserve. This occurred in figs 3.4 (A, B, C) for oil, figs 3.4 (D, E, F) for gas, all using model 4. Figs 3.5 (A, B, C) oil and 3.5 (D, E, F) gas for model 5. In model 6, figs 3.6 (A, B, C) oil and 3.6 (D, E, F) gas; in figs 3.7 (A, B, C) for oil and figs 3.7 (D, E, F) for gas for model 7, and in figs 3.9 (A, B, C) oil and 3.9 (D, E, F) gas for model 9.

Table 2.1: An Array of developed models by altering the input functions

Model No.	Input function	Cumulative Production $V_{(t)}$	Annual Production $P_{(t)}$	Peak Time (t_{pk})
1.	$\frac{1}{s}$	$R + (R-V_o)\, e^{-t/R}$	$\left(\frac{V_0}{R} - 1\right) e^{-\frac{t}{R}}$	at t_{pk}: $V_0 = R$
2.	$\frac{1}{S^2}$	$(t-R)\, R + (V_0 + R^2) e^{-t/R}$	$R - \frac{1}{R}(V_0 + R^2)\, e^{-t/R}$	at t_{pk}: $V_0 = -R^2$
3.	$\frac{1}{s + \frac{1}{R}}$	$(V_0 + t)\, e^{-t/R}$	$-\frac{1}{R}\,(V_0 + t)\, e^{-t/R}$	$-V_0$
4.	$\frac{1}{s\left(s + \frac{1}{R}\right)}$	$R^2(1-e^{-t/R}) + (V_0 - Rt)\, e^{-t/R}$	$\left(\frac{V_0}{R} - t\right) e^{-t/R}$	$Rt - R^2 - V_0 = 0$
5.	$\frac{1}{s\left(s + \frac{1}{R}\right)^2}$	$R^3(1-e^{-t/R}) + (V_0 - R^2 t - R\frac{t^2}{2})\, e^{-t/R}$	$\left(\frac{V_0}{R} - \frac{t^2}{2}\right) e^{-t/R}$	$Rt^2 - 2R^2t - 2V_0 = 0$
6.	$\frac{1}{s\left(s + \frac{1}{R}\right)^3}$	$R^4(1-e^{-t/R}) + (V_0 - R^3 t + \frac{R^2t^2}{2} - \frac{Rt^3}{6})\, e^{-t/R}$	$\left(\frac{V_0}{R} - \frac{t^3}{6}\right) e^{-t/R}$	$Rt^3 - 3R^2t^2 - 6V_0 = 0$
7.	$\frac{1}{s\left(s + \frac{1}{R}\right)^4}$	$R^5(1-e^{-t/R}) + (V_0 - R^4 t - R^3\frac{R^3t^2}{2} - \frac{R^2t^3}{6} - \frac{Rt^4}{24}) e^{-t/R}$	$\left(\frac{V_0}{R} - \frac{t^4}{24}\right) e^{-t/R}$	$Rt^4 - 2R^2t^3 - 24V_0 = 0$
8.	$\frac{1}{s\left(s + \frac{1}{R}\right)^n}$	$R^{n+1}(1-e^{-t/R}) + \left(V_0 - R^n t - R^{n-1}\frac{t^2}{2!} - \ldots - R^{n-(n-2)}\frac{t^{n-1}}{(n-1)!} - \frac{R^{(n-1)}t^n}{n!}\right) e^{-\frac{t}{R}}$	$\left(\frac{V_0}{R} - \frac{t^n}{n!}\right) e^{-t/R}$	$Rt^n - nRt^{2n-1} - (n!)V_0$
9.	$\frac{1}{s\left(s + \frac{1}{C}\right)}$	$RC + V_0 e^{-t/R} + \frac{RC}{R-C}\,(Ce^{-t/C} - Re^{-t/R})$	$\frac{RC}{R-C}(e^{-t/R} - e^{t/c}) - \frac{V_0}{R}$ $e^{t/c}$	$\frac{RC}{R-C}\; In\left[\frac{RC - V_0(R-C)}{R^2}\right]$

3. RESULT PRESENTATION AND DISCUSSION

3.1 Result presentation

The curve-fitting done with models in table 2.1 and data from Appendices A and B are shown here below as in figures 3.1a, 3.2a, 3.3a, 3.10. Also shown are figures 3.4 – 3.9 (A - F). However, there is no figure 3.8 (since model 8 is generalized model).

Curve Fittings

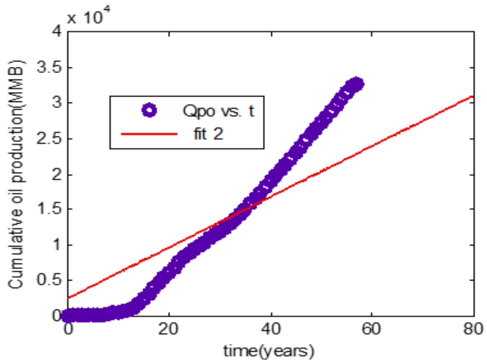


Fig 3.1A: Cumulative oil production versus time, R^2=0.7969. Model 1 does not fit, does not predict

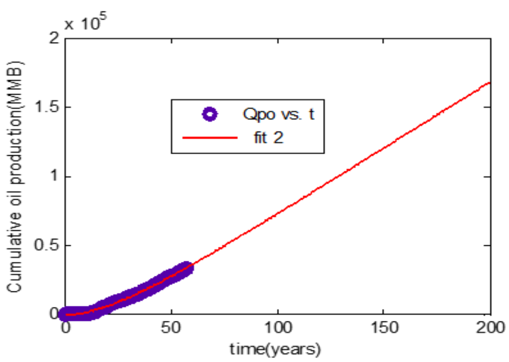


Fig3.2A: Cumulative oil production versus time, R^2=0.9978. Model 2 does not predict

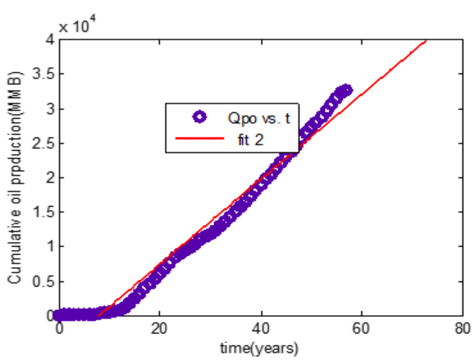


Fig 3.3A; Cumulative oil production versus time, R2=0.9708. Model 3 does not fit, does not predict

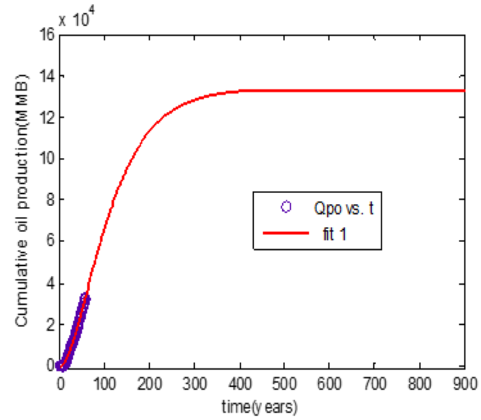


Fig 3.4A: Cumulative oil production versus time, R^2=0.9978

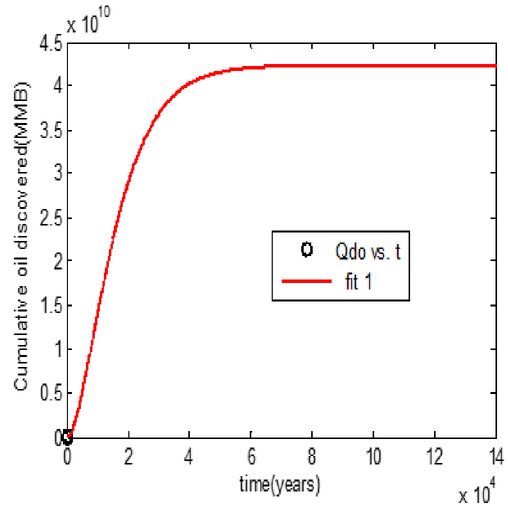


Fig 3.4B: Cumulative oil discovery versus time of discovery, R^2=0.9983

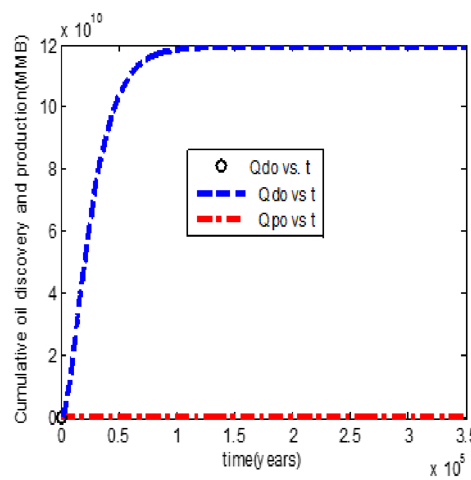


Fig 3.4C: Cumulative oil discovery and production versus time*

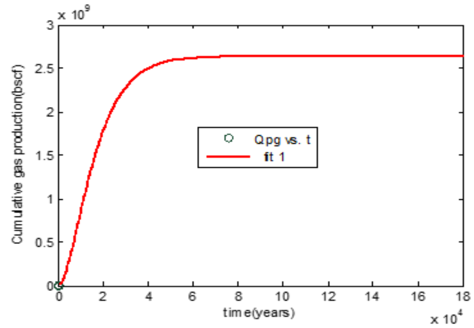


Fig 3.4D: Cumulative gas production versus time of production, R^2=0.9872

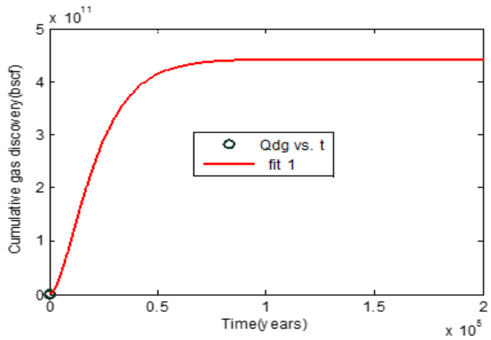


Fig 3.4E: Cumulative gas discovery versus R^2=0.9989

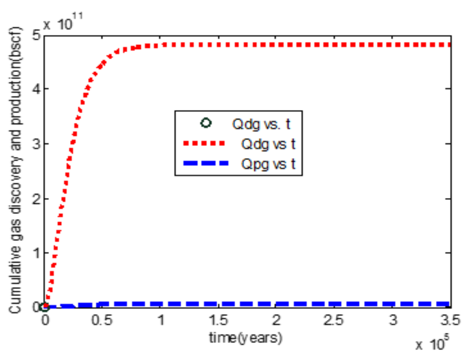


Fig 3.4F: Cumulative gas discovery and production versus time*

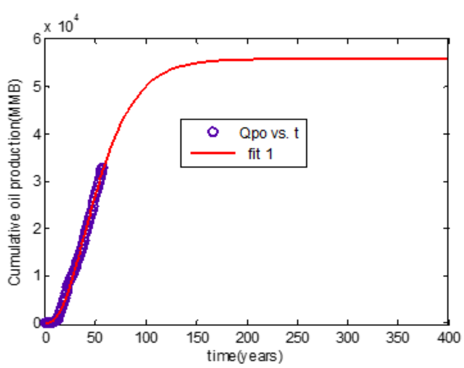


Fig 3.5A: Cumulative oil production versus time, R^2=0.9964

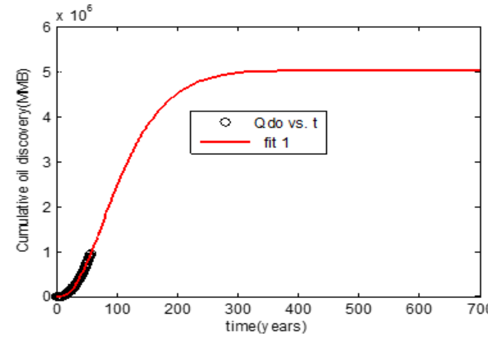


Fig 3.5B: Cumulative oil discovery versus time, R^2=0.9957

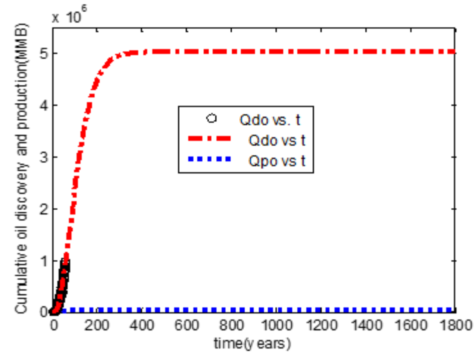


Fig3.5C: Cumulative oil discovery and production versus time*

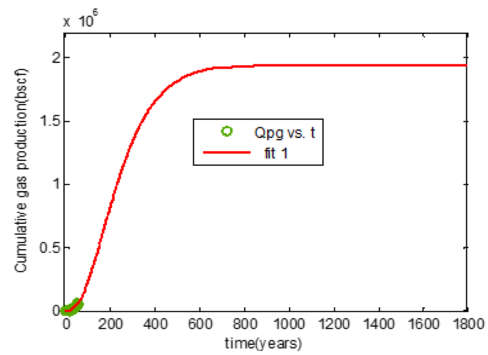


Fig 3.5D: Cumulative gas production versus time, R^2=0.9983

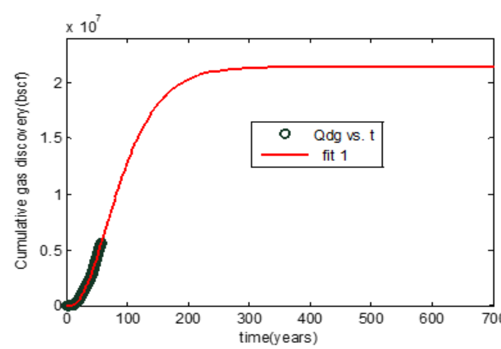


Fig 3.5E: Cumulative gas discovery versus time, R^2=0.9979

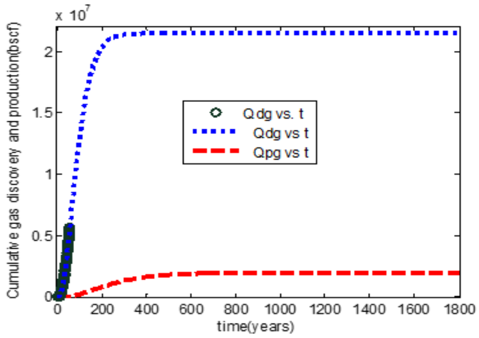


Fig 3.5F: Cumulative gas discovery and production versus time*

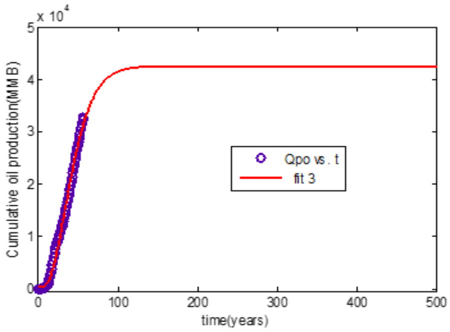


Fig 3.6A: Cumulative oil production versus time, R^2=0.9925

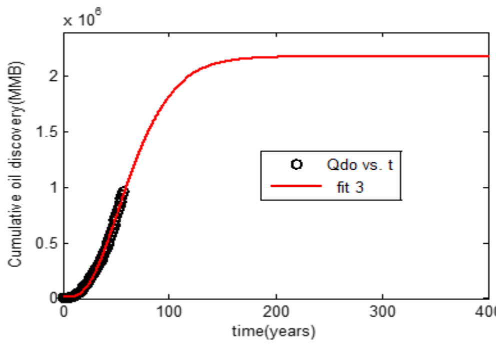


Fig 3.6B: Cumulative oil discovery versus time, R^2=0.9953

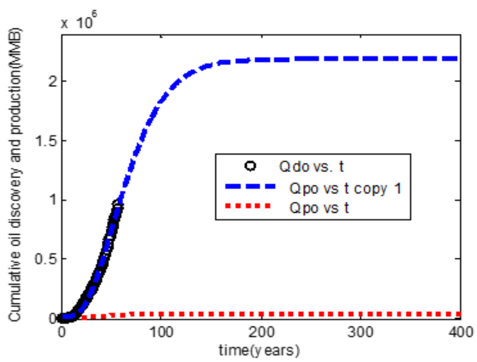


Fig 3.6C: Cumulative oil discovery and production versus time*

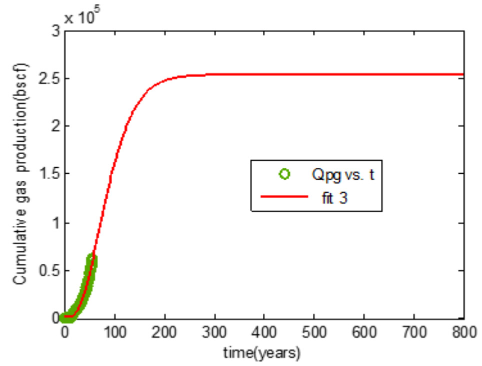


Fig 3.6D: Cumulative gas production versus time, R^2=0.9919

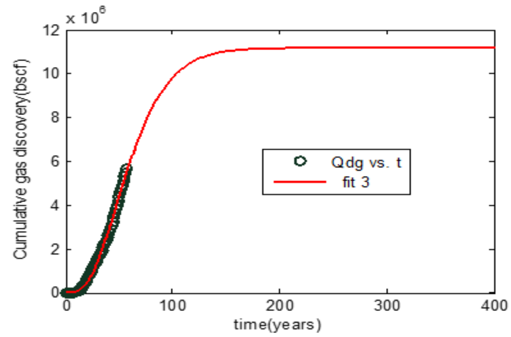


Fig 3.6E: Cumulative gas discovery versus time, R^2=0.9950

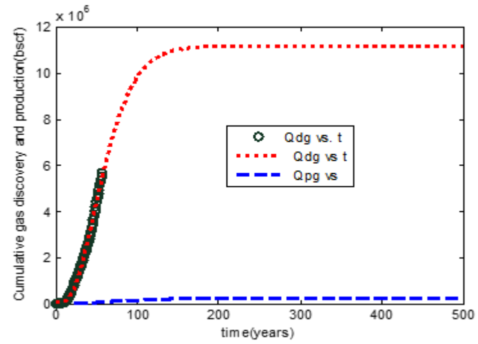


Fig 3.6F: Cumulative gas discovery and production versus time *

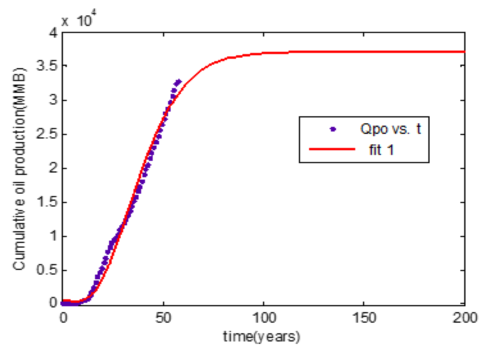


Fig 3.7A: Cumulative oil Production versus time, R^2=0.9872

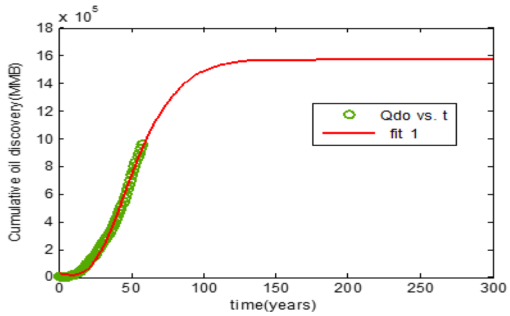


Fig 3.7B: Cumulative oil discovery versus time, R^2=0.9912

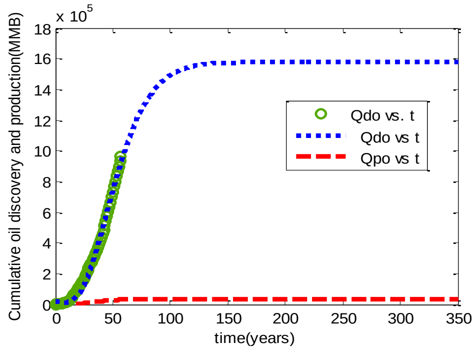


Fig 3.7C: Cumulative oil discovery and production versus time*

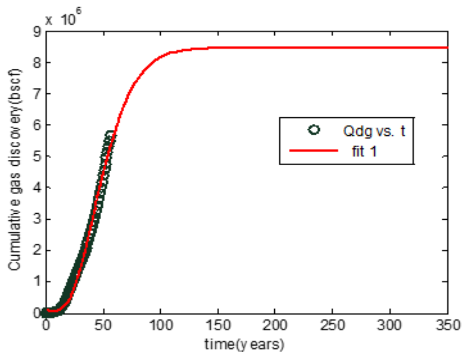


Fig 3.7D: Cumulative gas discovery versus time, R^2=0.9910

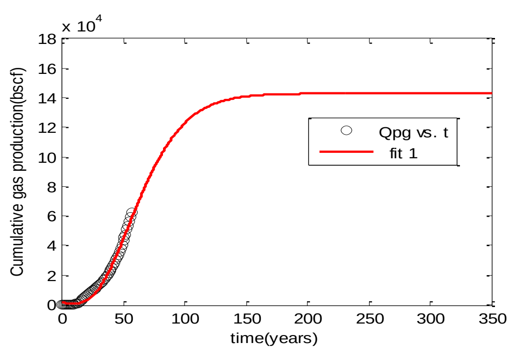


Fig 3.7E: Cumulative gas production versus time, R^2=0.9873

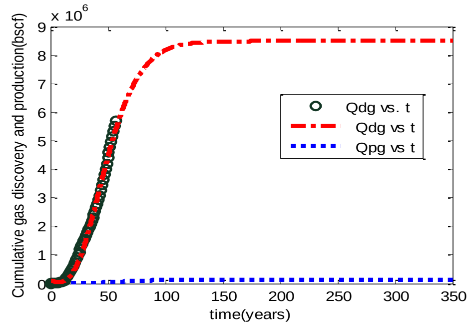


Fig 3.7F: Cumulative gas discovery and production versus time*

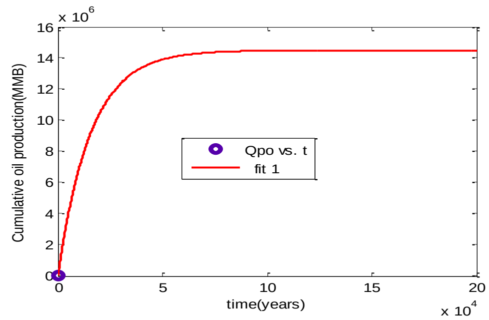


Fig 3.9A: Cumulative oil production versus time, R^2=0.9979, Model 9

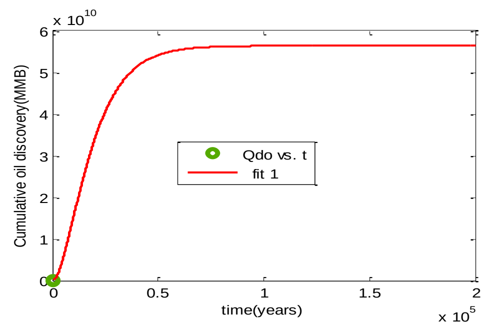


Fig 3.9B: Cumulative oil discovery versus time, R^2=0.9983, Model 9

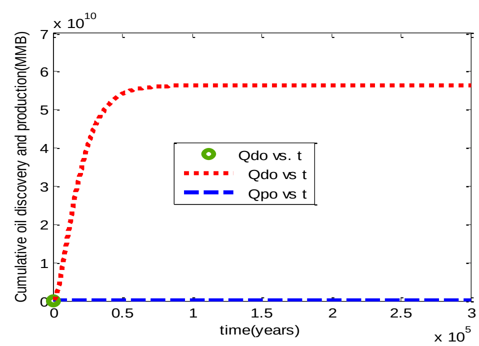


Fig 3.9C: Cumulative oil discovery and production versus time*, Model 9

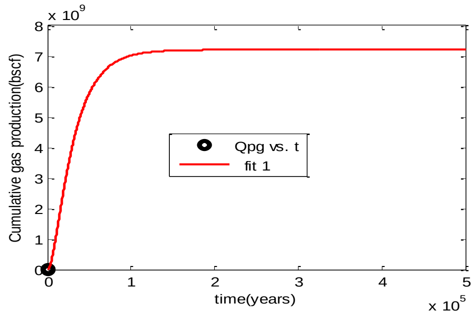


Fig 3.9D: Cumulative gas production versus time, R^2=0.9872, Model 9

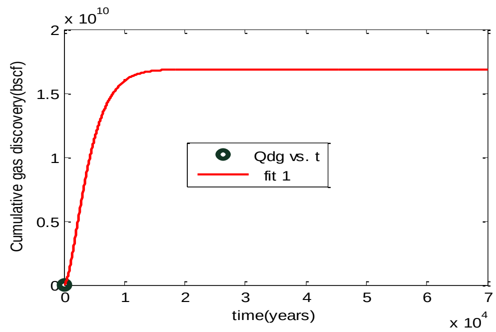


Fig 3.9E: Cumulative gas discovery versus time, R^2=0.9988, Model 9

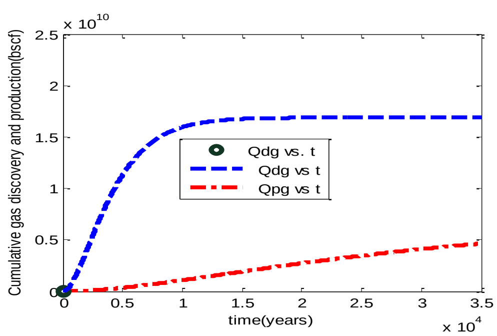


Fig 3.9F: Cumulative gas discovery and production versus time*,Model 9

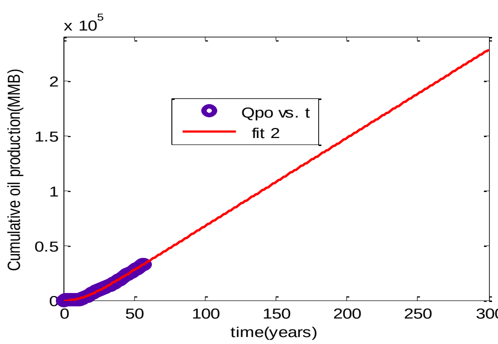


Fig 3.10; Cumulative oil production versus time, R^2=0.9979. Model 10 does not predict

3.2 Discussion of Result

All of models 1 through 3 have one problem or the other. For instance, a model may not fit the plot and will be discarded. For example, models 1 and 3. Sometimes, a model may plot but will not predict because it does not tapper out asymptotically parallel to x-axis, so that an ultimate value of y can read off (i.e sigmoidal profile). In this case, except a model is sigmoidal (i.e S-shaped), it cannot predict (Hubbert, 1956). Examples of those are figs 3.2a and 3.10; though these two figures fitted very well with R^2 of 0.9978 and 0.9979 respectively, they could not predict and so were discarded. Hence, for a model to be used, it must both fit and predict.

Figure 3.4 produced a rate plot with an apparent intersection of discovery and production of oil. The intersection obtained is $f(119{,}000) = 0$, which means the reserve will finish in 119,000 years from 1957. In the case of the gas, i.e rate plots of figs 3.4, the intersection occured at $f(151{,}000) = 0.895302$, i.e the gas reserve will finish in 151,000 years from 1957.

This is a good result except that in the model 4, gas have a negative initial reserve (V_O). In model 5, rate plots of figs 3.5 show that the intersection occurred at $f(725) = 0.136817$, giving us 725 years from 1957, just as the intersection of the gas section of the same model is $f(1194) = 1.62447$, i.e 1,194 years from 1957. This model is a good model because it has both positive initial reserve (V_O) as well as comparative high R^2.

In model 6, rate lots of figs 3.6 show that the intersection occurs at $f(523) = 0.0133776$, i.e 523 years from 1957. The gas part of the model, intersect at $f(483) = 0.000149588$, i.e 483 years from 1957. Again, this model is a good model for it has given us a positive initial reserve (V_O) even as its R^2 is not high as model 5.

In model 7, there is no duplicate plot since the two plots can be contained in one graph. The rate plot of fig 3.7 showed an intersection of $f(180) = 36.6086$ i.e 180 years from 1957. The gas counterpart of the model 7 intersected at $f(224) = 3.45806$, i.e 224 years from 1957. This model also gives positive initial reserve (V_O) but its R^2 is comparatively lower than that of model 5.

There is no model 8(or fig 3.8) result since it is a generalized model.

In model 9, which does not also have duplicate plot, rate plot of fig 3.9 gives an intersection of $f(5.87e^{+8}) = 0.485818$. This big value merely means that it is for a very long time while the intersection of the gas counterpart of model 9, i.e rate plot of fig 3.9 at $f(11880) = 178310$, i.e 11880 years from 1957. This model is not a good model. Like model 4, it has negative initial reserve (V_O) even as R^2 is promising to be a good one.

It is said that, in oil industry, oil is to be discovered before production. But in this case, not only that there is no oil is outside the reservoirs (in the rock), i.e the meaning of negative initial reserve.

In models 6 and 7, though naturally okay with positive initial reserve, like model 5 but their R^2 are not the best. In model 5, it meets the natural reality of positive initial reserve as well as best R^2 in the remaining models.

Again, it is in order because oil will likely finish before gas 725 years against 1194 year, i.e if this is put in the present dispensation, the oil will finish in the year 2682AD and gas 3151AD. These are indeed pretty long times.

4. CONCLUSSION

In this work, models were first developed from material balance of Nigerian petroleum around a composite reserve. Process control concepts were introduced to obtain the transfer functions so that as input functions are varied, new models are obtained, to note which input have the best impact. Hubbert oil depletion concept was employed for the peak determination. The Nigerian Petroleum Data were obtained from the Department of Petroleum Resources (DPR) of the Ministry of Petroleum and Minerals Resources, 7 Kofo Abayomi Street, Victoria Island, Lagos, as the experimental data for 57 years giving 57 data points. MatLab Package 7.9 version was employed in the mathematical computations and curve-fittings. From the curve-fitted plots, It is found that the Nigerian oil reserve will finish in the year 2682AD and the gas will follow sooth in the year 3151AD.

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